Exact resolution method for general 1D polynomial Schrödinger equation

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Corrigendum

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A few intermediate statements in this work (and also in reference [16] therein) are *mutually inconsistent* as they stand, and can be repaired as follows (keeping to the original equation numbers). They concern the integral

$$I_q(s,\lambda) \stackrel{\text{def}}{=} \int_q^{+\infty} (V(q')+\lambda)^{-s+1/2}\,\mathrm{d}q' \tag{12}$$
$$V(q) = +q^N + v_1 q^{N-1} + v_2 q^{N-2} + \cdots + v_{N-1} q$$

for which we overlooked some of the effects induced by a pole at $s = 0$.

We first correct a misprint in the expansion (16) and amend its notation, as

$$(V(q)+\lambda)^{-s+1/2} \sim \sum_\sigma \beta_\sigma(s) q^{\sigma-Ns} \qquad \text{for } q \to +\infty \quad \left(\sigma = \frac{N}{2}, \ \frac{N}{2}-1, \cdots\right) \tag{16$'$}$$

(the β_σ also depend on the parameters (v_j, λ), but this may remain implied). The inconsistencies then arise in the fully generic situation where $\beta_{-1}(s) \not\equiv 0$.

We selected a recessive solution $\psi_\lambda(q)$ of the Schrödinger equation, normalized according to its definition (11)–(12) through a symbolically defined integral,

$$\mathcal{I}(q,\lambda) = \int_q^{+\infty} (V(q')+\lambda)^{1/2}\,\mathrm{d}q'.$$

However, we switched between three specifications of $\mathcal{I}(q,\lambda)$ which we now distinguish by subscripts:

$$\mathcal{I}_{(0)}(q,\lambda) = \text{finite part of } I_q(s,\lambda)_{s\to 0} \tag{15}$$

$$\mathcal{I}_{(1)}(q,\lambda) \sim -\sum_{\sigma\neq 0} \beta_\sigma \frac{q^{\sigma+1}}{\sigma+1} - \beta_{-1}\log q \qquad \text{for } q \to +\infty \tag{17}$$

and

$$\begin{aligned} \mathcal{I}_{(2)}(0,\lambda) &= -\frac{1}{2}[\partial_s Z_{\mathrm{cl}}(s,\lambda)]_{s=0} \\ \mathcal{I}_{(2)}(q,\lambda) &= \mathcal{I}_{(2)}(0,\lambda) - \int_0^q (V(q')+\lambda)^{1/2}\,\mathrm{d}q'. \end{aligned} \tag{26}$$

We implied $\mathcal{I}_{(0)} = \mathcal{I}_{(1)} = \mathcal{I}_{(2)}$ from the case $\beta_{-1}(s) \equiv 0$ where that coincidence occurs; instead, the most general relationship involves additive constants, obtainable by explicitly computing suitable finite parts:

$$\mathcal{I}_{(0)} + \frac{2(1-\log 2)}{N}\beta_{-1}(0) = \mathcal{I}_{(1)} - \frac{2\log 2}{N}\beta_{-1}(0) + \frac{1}{N}\partial_s\left[\frac{\beta_{-1}(s)}{1-2s}\right]_{s=0} = \mathcal{I}_{(2)}. \tag{$*$}$$

Thus, the factor $e^{\mathcal{I}(q,\lambda)}$ within the solution $\psi_\lambda(q)$ retained several normalizations in full generality, and, essentially: $\psi_\lambda(q)$ *obeys the (Sibuya) asymptotic formula (19) under* $\mathcal{I}(q,\lambda) \stackrel{\text{def}}{=} \mathcal{I}_{(1)}(q,\lambda)$, whereas it *obeys the basic identities (35) under* $\mathcal{I}(q,\lambda) \stackrel{\text{def}}{=} \mathcal{I}_{(2)}(q,\lambda)$ (while $\mathcal{I}_{(0)}$ becomes useless).

Conversion factors deduced from equation $(*)$ now allow consistent reformulations.

At the same time, the quantity $\psi_\lambda(q)$ never served in isolation in this work whose only cornerstone is the *Wronskian* of the pair of solutions $(\psi_\lambda, \psi_\lambda^{[1]})$, which is *insensitive* to this particular ambiguity simply because these two solutions carry *opposite* coefficients $\beta_{-1}(s)$. Therefore, the main core of our article remains unaffected, especially the main functional relation (40) and all derived results.

By contrast, further uses of the exact quantization formalism (cf. [1]) are seriously compromised if those inconsistencies are not cured, and we consequently apologise to readers for this oversight.

Reference

[1] Voros A 2000 Exercises in exact quantization *J. Phys. A: Math. Gen.* **33** at press (Voros A 2000 *Preprint* math-ph/0005029)

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